Dynamics of planetary systems in the NEAT context: stability and ephemeris prediction

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Dynamical perturbations in planetary systems?

• **Goals:**
  - Investigating stability of candidate planetary systems
  - Predicting ephemeris: over which time basis?

• **Keplerian model:**
  - Stability ensured
  - Ephemeris easy to predict
  - But: planets perturb themselves

• **Perturbations among planets:**
  - Secular parturbations: can be resonant or non-resonant; affect ephemeris (~time-scale), but usually do not affect stability.
  - Chaotic perturbations and diffusion: affect both; can lead to instability

• **In principle, one should take everything into account, but this requires the use of sophisticated codes. It is always necessary?**

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Perturbations in planetary systems: time-scales

- **Secular planetary perturbations time-scale**
  - Make orbits precess and eccentricity fluctuate: affects ephemeris!
  - Solar system: $\sim 10^4 - 10^5$ years
  - More compact planetary system: hundreds of years!
  - Worse if the system is resonant. Example: Gl 876 $\Rightarrow \sim 5$ years

- **Analytics:**
  - Consider a small mass planet perturbed by an outer, more massive one. To lowest order, the orbital precession time-scale reads

$$\frac{t_{\text{prec}}}{P_{\text{orb}}} \approx \frac{4}{3} \frac{M_*}{m_p} \left( \frac{a}{a_p} \right)^3$$

  - Application 1: Earth / Jupiter $\Rightarrow t_{\text{prec}} \approx 10^5$ years
  - Application 2: Gliese 581 $\Rightarrow t_{\text{prec}} \approx 1000$–10000 years
  - Can be much shorter in some cases
Perturbations in planetary systems: strategy

• Chaotic perturbations leading to instability
  – The closer the planets, the more rapid the instability
  – Can occur after several thousands of years ⇒ need an accurate code to explore the dynamics

• Conclusion:
  – For low accuracy ephemeris over 10 — 20 years for « regular » systems, Keplerian approximation is enough
  – For high accuracy ephemeris or resonant systems, planetary perturbations need to be taken into account
  – Analytical formulas depend on whether systems are resonant or not ⇒ numerical integration preferred when nature is unknown
  – For stability analysis: Fast and accurate codes required ⇒ symplectic N-body schemes

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Symplectic N-body codes (1)

- Numerical integration = finding $q(t)$ once $q(t-\tau)$ is known
- Classical methods do not keep $\mathcal{H} = \text{Hamiltonian}=\text{cte}$ (Runge-Kutta, Burlish & Stoer …)
- A symplectic scheme exactly preserves $\mathcal{H}$ or a nearby one
- The global error is bound
- Interesting for long term stability
Symplectic N-body codes (2)

• Basic hypotheses of symplectic integration: \( H = H_A + H_B \)
  where we know how to exactly integrate \( H_A \) and \( H_B \)

• We usually assume \( H_B \ll H_A \left( \frac{H_B}{H_A} \sim \epsilon \right) \)

• First method: Advancing \( H_A \) during \( \tau \) and then \( H_B \) during \( \tau \), we build a first order symplectic integrator: we exactly solve a Hamiltonian \( H_{\text{integ}} = H + O(\tau) \)

• Second method: Integrating 1) \( H_A \) during \( \tau/2 \), \( H_B \) during \( \tau \), 3) \( H_A \) during \( \tau/2 \), we build a second order symplectic integrator: In that case \( H_{\text{integ}} = H + O(\tau^2) \)

• Thanks to \( \left( \frac{H_B}{H_A} \sim \epsilon \right) \), we even have

\[ \Rightarrow \text{We can integrate with a large timestep} \quad H_{\text{integ}} = H + O(\epsilon \tau^2) \]
Practical N-body symplectic schemes

• **Popular method:** Wisdom-Holman Mapping
  - Hypothesis: \((m_i \ll m_0 \text{ for every } i = 1, \ldots, n-1)\) (planetary systems)
  - \(H_A\) = Sum of Keplerian Hamiltonians (we can integrate…)
  - \(H_B\) = What’s left = Mutual perturbations ⇒ function of \(x_i\)’s only

• The condition \(H_B \ll H_A\) is fulfilled if
  1) Body #0 is much more massive than the others
  2) Mutual distances are not too small (no close encounters)
Close encounters?

- Occurs when two bodies come close to each other. We have strong and short scale perturbations. Condition (2) is no longer fulfilled.

- The best to do: reduce the timestep when this occurs.

- But keeping a symplectic integrations requires a constant timestep! Two ways to solve the problem:
  - We drop symplecticity for a while during the time of the encounter (RMVS code, Levison & Duncan, public). Ok from a statistical point of view.
  - We change the $H_A + H_B$ splitting during the close encounter. We keep symplecticity but the computing time is longer (MERCURY code Chambers, public)
  - We split potentials in concentric spherical shells and attribute one timestep to each shell (SyMBA code Duncan, Lee & Levison). The one to use to test stability! We have it
Example of application: Gliese 581
(Beust et al. 2008, Mayor et al. 2009)

- Computed using the symplectic integrator SyMBA (Lee et al. 1998)
- Temporal evolution of the eccentricities of the planets over $10^4$ yr.
- The dynamics is very regular
Long term integration of Gliese 581

- Over $10^8$ yr, it is still regular
- Given the precession frequencies (much shorter than in the Solar system), $10^8$ yr is equivalent to several Gyr for the Solar System.
- There is no resonance
- Conclusion: the system is stable

<table>
<thead>
<tr>
<th>Planet</th>
<th>Semi-major axis</th>
<th>Eccentricity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gl 581 b</td>
<td>0.040609 - 0.046185</td>
<td>0.01 - 0.095</td>
</tr>
<tr>
<td>Gl 581 c</td>
<td>0.072885 - 0.073</td>
<td>0.07 - 0.16</td>
</tr>
<tr>
<td>Gl 581 d</td>
<td>0.2522 - 0.2528</td>
<td>0.12 - 0.1246</td>
</tr>
</tbody>
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Maximum variation ranges of orbital elements over $10^8$ yr
Stability analysis: increase masses ($\propto 1/\sin i$)

- We plot the maximum variation ranges of semi-major axis as a function of inclination.

- Conclusion: the system is stable up to $i \sim 40^\circ$