Dynamics of planetary systems in the NEAT context : stability and ephemeris prediction

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Dynamical perturbations in planetary systems ?

- Goals :
  - Investigating stability of candidate planetary systems
  - Predicting ephemeris : over which time basis ?
- Keplerian model :
  - Stability ensured
  - Ephemeris easy to predict
  - But : planets perturb themselves
- Perturbations among planets :
  - Secular parturbations : can be resonant or non-resonant; affect ephemeris (~time-scale), but usually do not affect stability.
  - Choatic perturbations and diffusion : affect both; can lead to instability
- In principle, one should take everything into account, but this requires the use of sophisticated codes. It is always necessary ?

Perturbations in planetary systems : time-scales

- Secular planetary perturbations time-scale
  - Make orbits precess and eccentricity fluctuate : affects ephemeris !
  - Solar system : ~ $10^4 - 10^5$ years
  - More compact planetary system : hundreds of years !
  - Worse if the system is resonant. Example : Gl 876 ⇒ ~ 5 years
- Analytics:
  - Consider a small mass planet perturbed by an outer, more massive one. To towest order, the orbital precession time-scale reads

$$\frac{t_{prec}}{P_{orb}} \approx \frac{4}{3}\frac{M_*}{m_p}\left(\frac{a}{a_p}\right)^3$$

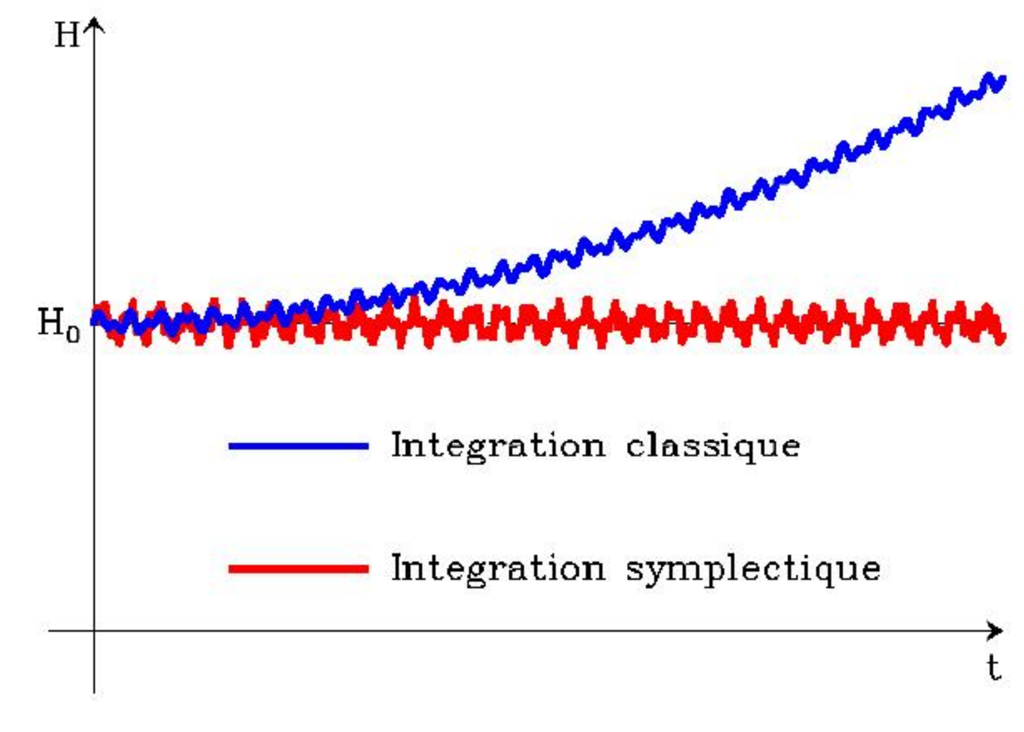
  - Application 1 : Earth / Jupiter ⇒ $t_{prec} \approx 10^5$ years
  - Application 2 : Gliese 581 ⇒ $t_{prec} \approx 1000$– 10000 years
  - Can be much shorter in some cases

Perturbations in planetary systems : strategy

- Chaotic perturbations leading to instability
  - The closer the planets, the more rapid the instability
  - Can occur after several thousands of years ⇒ need an accurate code to explore the dynamics
- Conclusion:
  - For low accuracy ephemeris over 10 — 20 years for « regular » systems, Keplerian approximation is enough
  - For high accuracy ephemeris or resonant systems, planetary perturbations need to be taken into account
  - Analytical formulas depend on whether systems are resonant or not ⇒ numerical integration preferred when nature is unknown
  - For stability analysis : Fast and accurate codes required ⇒ symplectic N-body schemes

Symplectic N-body codes (1)

- Numerical integration = finding *q(t)* once *q(t-τ)* is known
- Classical methods do not keep *H= Hamiltonian*=cte (Runge-Kutta, Burlish & Stoer …)
- A symplectic scheme exactly preserves H or a nearby one
- The global error is bound
- Interesting for long term stability



Symplectic N-body codes (2)

- Basic hypotheses of symplectic integration : $H = H_A + H_B$ where we know how to exactly integrate H_A and H_B
- We usually assume $H_B << H_A$ $\left(H_B / H_A \sim \varepsilon\right)$
- First method : Advancing H_A during τ and then H_B during τ, we build a first order symplectic integrator : we exactly solve a Hamiltonian $H_{\text{integ}} = H + O(\tau)$
- Second method : Integrating 1) H_A during τ/2, H_B during τ, 3) H_A during τ/2, we build a second order symplectic integrator In that case $H_{\text{integ}} = H + O\left(\tau^2\right)$
- Thanks to $\left(H_B / H_A \sim \varepsilon\right)$, we even have

⇨ We can integrate with a large timestep $H_{\text{integ}} = H + O\left(\varepsilon\tau^2\right)$

Practical N-body symplectic schemes

- Popular method: Wisdom-Holman Mapping
  - Hypothesis : ($m_i \ll m_0$ pour every $i = 1,\ldots,n-1$) (planetary systems)
  - H_A = Sum of Keplerian Hamiltonians (we can integrate…)
  - H_B = What's left = Mutual perturbations $\Rightarrow$ function of x_i's only)

- The condition $H_B \ll H_A$ is fulfilled if
  1) Body #0 is much more massive than the others
  2) Mutual distances are not too small (no close encounters)

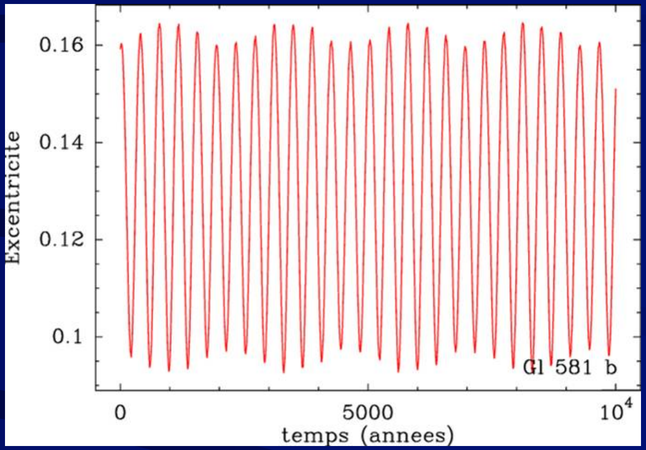
Close encounters ?

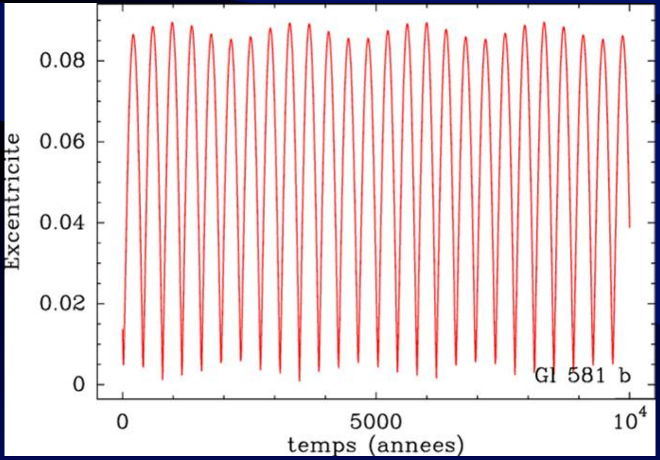
- Occurs when two bodies come close to each other. We have strong and short scale perturbations. Condition (2) is no longer fulfulled.
- The best to do : reduce the timestep when this occurs.
- But keeping a symplectic integrations requires a constant timestep ! Two ways to solve the problem:
  - We drop symplecticity for a while during the time of the encounter(RMVS code , *Levison & Duncan, public*). Ok from a statistical point of view.
  - We change the H_A+H_B splitting during the close encounter. We keep symplecticity but the computing time is longer (MERCURY code *Chambers, public*)
  - We split potentials in concentric spherical shells and attribute one timestep to each shell (SyMBA code *Duncan, Lee & Levison*). *The one to use to test stability ! **We have it***

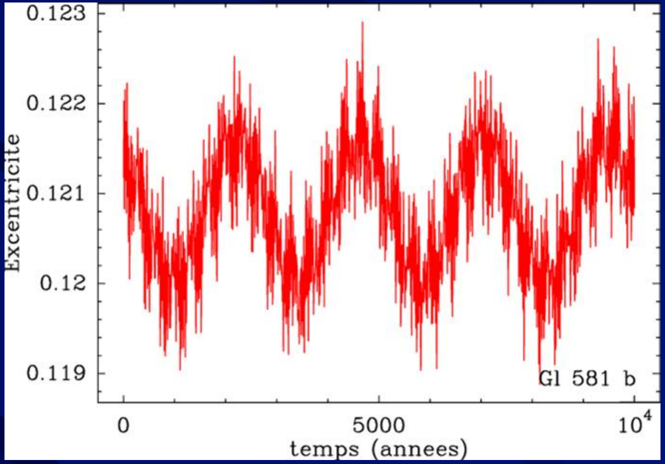
Example of application : Gliese 581

(Beust et al. 2008, Mayor et al. 2009)

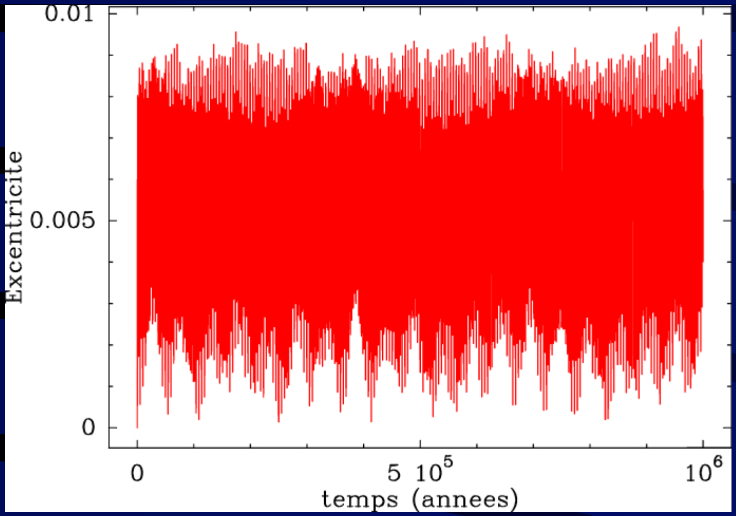
- Computed using the symplectic integrator SyMBA (Lee et al. 1998)
- Temporal evolution of the eccentricities of the planets over 10^4 yr.
- The dynamics is very regular







Long term integration of Gliese 581



Gl 581 c

- Over 10^8 yr, it is still regular
- Given the precession frequencies (much shorter than in the Solar system), 10^8 yr is equivalent to several Gyrs for the Solar System.
- There is no resonance
- Conclusion : the system is stable

Maximum variation ranges of orbital elements over 10^8 yr

Planet	Semi-major axis	Eccentricity
Gl 581 b	0.040609 - 0.046185	0.01 - 0.095
Gl 581 c	0.072885 - 0.073	0.07 - 0.16
Gl 581 d	0.2522 - 0.2528	0.12 - 0.1246

Stability analysis : increase masses (∝1/sin i)

- We plot the maximum variation ranges of semi-major axis as a function of inclination.
- Conclusion : the system is stable up to i ~ 40°