Radial velocities *vs* Astrometry

- Consider an explanet with mass m_p orbiting a star of mass M_* at distance d, with semi-major axis a, period P, eccentricity e, inclination i with respect to the sky.
- Radial velocity wobble :

$$V_r = \underbrace{\frac{m_p}{m_p + M_*} \frac{2\pi}{P} \frac{a \sin i}{\sqrt{1-e^2}}}_{K} \left(\cos(\omega + v) + e \cos \omega \right) \quad \propto \quad a^{-1/2}$$

$$K = \sqrt[3]{\frac{2\pi G}{P}} \frac{m_p \sin i}{\left(m_p + M_*\right)^{2/3}} \frac{1}{\sqrt{1-e^2}} \approx 28.4 \text{ m s}^{-1} \times \left(\frac{P}{1 \text{ an}}\right)^{-1/3} \left(\frac{m_p \sin i}{M_{\text{Jup}}}\right) \left(\frac{M_*}{M_\odot}\right)^{-2/3}$$

- Astrometric displacement

$$\Delta\alpha = -\frac{m_p}{m_p + M_*} \frac{a}{d} \frac{\left(1-e^2\right)}{1 + e \cos v} \sqrt{1 - \sin^2(\omega + v) \sin^2 i} \sim \frac{m_p}{M_*} \frac{a}{d} \quad \propto \quad a$$

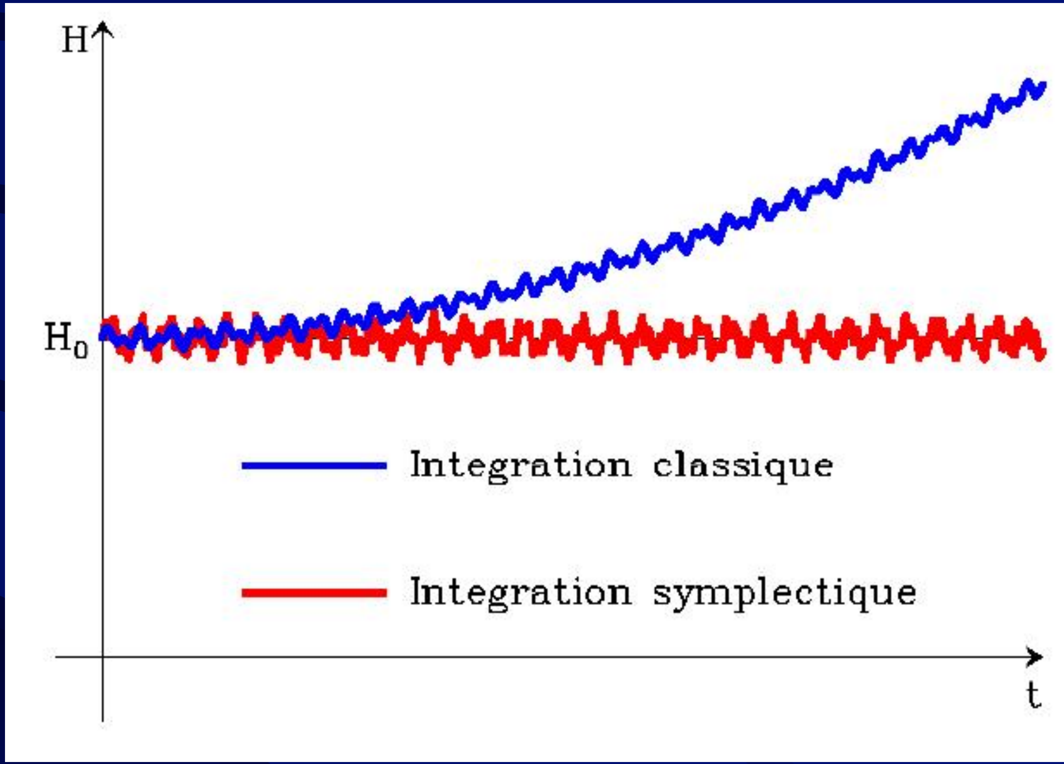
- Astrometry is sensitive to large periods, while velocimetry is sensitive to close-in planets.
- But it is best for nearby stars

Observing nearby stars

- Requirement for astrometry : be patient ! (long periods)
- But for a nearby star, we can detect shorter period planets.
- Goal with such systems: Detecting solar system analogs.
- Issues to be investigated:
  - How frequent are solar-like systems around solar type stars ?
  - Do they all scale the same way ?
  - What about planets in binary systems ?
- Dynamical issues :
  - Orbital evolution in detected systems : stability, unseen planets ?
  - Perturbations by companions ⇒ Study with symplectic codes

Symplectic N-body codes (1)

- Numerical integration = finding *q(t)* once *q(t-τ)* is known
- Classical methods do not keep *H= Hamiltonian*=cte (Runge-Kutta, Burlish & Stoer …)
- A symplectic scheme exactly preserves H or a nearby one
- The global error is bound
- Interesting for long term stability



Symplectic N-body codes (2)

- Basic hypotheses of symplectic integration : $H = H_A + H_B$ where we know how to exactlty integrate H_A and H_B. We also try to have
- On suppose en plus $H_B << H_A$ $\left(H_B / H_A \sim \varepsilon \right)$
- We have

$$\exp(\tau A).\exp(\tau B) = \exp\left[\tau(A+B) + O(\tau^2)\right]$$

⇨ Advancing H_A during τ and then H_B during τ, we build a first order symplectic integrator : we exactly solve a Hamiltonian

$$H_{\text{integ}} = H + O(\tau)$$

Symplectic N-body codes (3)

- We also have

$$\exp\left(\frac{\tau}{2}B\right)\exp(\tau A)\exp\left(\frac{\tau}{2}B\right)=\exp\left[\tau(A+B)+O(\tau^3)\right]$$

⇨ We build a second order symplectic integrator if we integrate 1) H_A during τ/2, H_B during τ, 3) H_A during τ/2. In this case

$$H_{\text{integ}}=H+O(\tau^2)$$

- There are also 4, 6, 8… order methods
- In fact, if $H_B/H_A \sim \varepsilon$, we even have

$$H_{\text{integ}}=H+O(\varepsilon\tau^2)$$

⇨ We can integrate with a large timestep

Practical N-body symplectic schemes

- Simplest method : $T+U$
  - H=Hamiltonian of N body problem = Kinetic + Potenetial = $T+U$
  - Canonically conjugate variables : $x_i=x,y,z..$ $p_i=mv_x,mv_y,mv_z...$
  - $T = T(p_i)$ and $U=U(x_i)$ $\Rightarrow$ We can integrate separately T and U
  - $\Rightarrow$Symplectic method with $H_A=T$, $H_B=U$. But we don't have $H_B<<H_A$
- Second exemple: MVS method (Mixed Variable Symplectic)
  - Hypothesis : ($m_i \ll m_0$ pour every $i = 1,...,n-1$) (planetary systems)
  - H_A = Sum of Keplerian Hamiltonians (we can integrate…)
  - H_B = What's left = Mutual perturbations $\Rightarrow$ function of x_i's only)
- The condition $H_B \ll H_A$ is fulfilled if

  1) Body #0 is much more massive than the others

  2) Mutual distances are not too small (no close encounters)

Close encounters ?

- Occurs when two bodies come close to each other. We have strong and short scale perturbations. Condition (2) is no longer fulfulled.
- The bes to do : reducing the timestep when this occurs.
- But keeping a symplctic integrations requires a constant timestep ! Two ways to solve the problem:
  - We drop symplecticity for a while during the time of the encounter (RMVS code , *Levison & Duncan*). Ok from a statistical point of view.
  - We change the H_A+H_B splitting during the close encounter. We keep symplecticity but the computing time is longer (MERCURY code *Chambers*)
  - We split potentials in concentric spherical shells and attribute one timestep to each shell (SyMBA code *Duncan, Lee & Levison*).

The HJS method (*Hierarchical Jacobi symplectic, Beust 2003*)

- The splitting of H is done according to the hierarchical structure of the stellar system.
- H_A = Sum of Keplerian Hamiltonians corresponding to centers of mass orbiting other centers of mass
- H_B = What's left
- The condition $H_B \ll H_A$ is fulfilled irrespective of the various masses, once the nested orbits are very different in size.

Analysis of the astrometric signal

- Issue : detecting an astrometric wobble of a few μas
- Perturbation sources (for a star at 50 pc) :

Proper motion over 1 period	0.1 μas – 1 mas
Non linear proper motion on sky	0.1 mas after 30 yrs
Parallactic orbit	20 mas
Effect of Earth eccentricity	0.3 mas
Barycentric motion of the Sun	0.15 mas
Planetary perturbations on the Earth	0.03 mas after 30 yrs

- ⇒ Need for an accurate modeling of all these effects !